

A sensitivity analysis for nonrandomly missing categorical data arising from a national health disability survey

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SUMMARY

Using data from 145 007 adults in the Disability Supplement to the National Health Interview Survey, we investigated the effect of balance difficulties on frequent depression after controlling for age, gender, race, and other baseline health status information. There were two major complications: (i) 80% of subjects were missing data on depression and the missing-data mechanism was likely related to depression, and (ii) the data arose from a complex sample survey. To adjust for (i) we investigated three classes of models: missingness in depression, missingness in depression and balance, and missingness in depression with an auxiliary variable. To adjust for (ii) we developed the first linearization variance formula for nonignorable missing-data models. Our sensitivity analysis was based on fitting a range of ignorable missing-data models along with nonignorable missing-data models that added one or two parameters. All nonignorable missing-data models that we considered fit the data substantially better than their ignorable missing-data counterparts. Under an ignorable missing-data mechanism, the odds ratio for the association between balance and depression was 2.0 with a 95% CI of (1.8, 2.2). Under 29 of the 30 selected nonignorable missing-data models, the odds ratios ranged from 2.7 with 95% CI of (2.3, 3.1) to 4.2 with 95% CI of (3.9, 4.6). Under one nonignorable missing-data model, the odds ratio was 7.4 with 95% CI of (6.3, 8.6). This is the first analysis to find a strong association between balance difficulties and frequent depression.

Keywords: Balance; Complex sample surveys; Depression; Ignorable missing-data mechanism; Missing at random; Nonignorable missing-data mechanism; Selection model.

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1. INTRODUCTION

The ability to maintain balance is essential to nearly all activities of daily living. People with chronic balance disorders are significantly disabled in many day-to-day functions, particularly those that require stabilizing the body during weight-shifting, bending, or rapid head motion. Unfortunately, balance problems are widespread and have serious consequences. The National Institute on Deafness and Other Communication Disorders (NIDCD) estimates that 12.5 million Americans over the age of 65 have dizziness or balance problems that significantly interfere with their lives (NIDCD, 1995). Because balance is normally an unconscious process, patients often have difficulty articulating their symptoms, and physicians can have difficulty identifying the problem and determining its cause. Consequently, little is known about the association of balance problems with other health outcomes.

One of our major interests was whether or not balance problems are associated with frequent medically treated depression. For public health reasons, our emphasis is on depression that requires medical treatment. It is thought that chronic balance/dizziness problems and psychological outcomes are related because both imbalance/dizziness and negative affectivity (a psychological trait with high levels of anxiety and depression) can arise from a common neurophysiological mechanism Hudson and Pope (1994). Recent population-based surveys (Grimby and Rosenhall, 1995; Honrubia *et al.*, 1996; Yardley *et al.*, 1998) found that dizzy subjects reported more depressive symptoms than non-dizzy subjects. However, Tinetti *et al.* (2000) found only 'marginal significance' ($P = 0.085$) and Nazareth *et al.* (1999) excluded anxiety as nonsignificant in a stepwise logistic regression for factors associated with dizziness. Moreover, these conclusions can only be regarded as tentative because of the specialized nature of the populations associated with these studies.

To further investigate the relationship between balance impairment/dizziness and frequent depression, we analysed data from the Disability Supplement to the National Health Interview Survey (NHIS-D), the first multipurpose and nationally representative disability survey conducted in the United States. The objective of the survey was to investigate the prevalence and impact of disability in the US non-institutionalized civilian population. In 1994 and 1995, the core questionnaire probing demographic characteristics and general health information was administered to all members of the sampled household.

In addition, the Phase 1 Disability Questionnaire collected basic data on disability. One of the Phase 1 questionnaire topics was sensory impairments, which included questions on chronic imbalance and dizziness. Because both chronic imbalance/dizziness and depression are not prevalent among children, our analysis was restricted to the 145 007 adults (age 18 years or older) surveyed in NHIS-D. Information on chronic imbalance/dizziness, defined as balance or dizziness problems lasting for at least 3 months, was collected in Phase 1 and was available from 141 960 adults (with 3047 or 2.1% missing due to refusals or don't knows).

In a somewhat nonstandard design, inclusion in Phase 2 was based on responses to a large series of Phase 1 questions related to disability. These include sensory limitations (e.g. trouble seeing, trouble hearing), dizziness, balance limitations, physical limitations (e.g. uses walker, uses cane), trouble with mental functions (e.g. frequently depressed or anxious, paranoia), selected disability conditions (e.g. cerebral palsy, autism), services (e.g. physical therapy, occupational therapy), and perceived disability. Only the 29 019 adult Phase 1 participants who answered 'yes' to at least one of these questions were included in Phase 2 (Table 1). Importantly, inclusion in Phase 2 did not depend on the sampling of the investigators as would have occurred if investigators randomly selected with nonzero probabilities some subjects who answered 'yes' to at least one question and some subjects who answered 'no' to all the questions. Subsequently, adult participants in Phase 2 were asked if they experienced frequent depression with medical treatment and 25 614 provided answers. Thus the total fraction of adults who provided a response to the Phase 2 question on frequent medically treated depression was $25\,614/145\,007 = 18\%$. The major goal of our analysis was to estimate the association between balance/dizziness and frequent

Table 1. Data on depression and balance

Answered yes to at least one		Included in Phase 2 not missing ($R = 0$)		Not included in Phase 2 missing ($R = 1$)		
Phase 1 question	balance problem	$Y = 0$ no	$Y = 1$ yes	$Y = 0$ no	$Y = 1$ yes	total
yes ($Z = 0$)	no ($X = 0$)	17 677	4685	0	0	
	yes ($X = 1$)	1 768	984	0	0	
no ($Z = 1$)	no ($X = 0$)	0	0	?	?	115 651
	yes ($X = 1$)	0	0	?	?	1 195

The zeros are structural.

medically-treated depression with a sensitivity analysis to investigate the effects of the missing-data mechanism and an adjustment for the variance to account for the complex survey design.

2. LIKELIHOOD FORMULATIONS

To fix ideas, we derive the likelihood for a simple example with only one covariate, balance. Extensions to more covariates are straightforward. Let $X = 1$ if a subject had balance problems and 0 otherwise. Let $Z = 1$ if the subject were included in Phase 2 by answering ‘yes’ to one of the appropriate questions, and 0 otherwise. Let $Y = 1$ if medically treated depression on Phase 2 and 0 otherwise. Let $R = 1$ if missing medically treated depression on Phase 2 and 0 otherwise.

Missing only depression

Suppose we could observe n_{zxy} , the number of subjects with variable z for inclusion in Phase 2, balance at level x , and depression outcome y . Also suppose we could observe w_{zx} , the number of subjects missing data on depression with variables z for inclusion and balance level at x . Let $\text{pr}(R = 1 | z, x; \zeta)$ denote missing-data model and let $\text{pr}(y | z, x; \pi)$ denote the model of interest. The likelihood kernel is

$$L_0 = \prod_x \prod_y \prod_z [\text{pr}(R = 0 | z, x; \zeta) \text{pr}(y | z, x; \pi)]^{n_{zxy}} \prod_x \prod_z [\sum_y \text{pr}(R = 1 | z, x; \zeta) \text{pr}(y | z, x; \pi)]^{w_{zx}} \quad (1)$$

Because $\text{pr}(R = r | Z = z, x; \zeta)$ factors from the likelihood and ζ is distinct from π (i.e. no parameter in ζ is a function of any parameter in π , or vice versa), the missing-data mechanism is ignorable (Rubin, 1974, 1976). It would therefore appear that one could adequately base inference about π on $\Pi_x \Pi_y \Pi_z \text{pr}(y | z, x; \pi)^{n_{zxy}}$. However, due to the special nature of z , such inference is problematic. The problem arises because $n_{1xy} = w_{0x} = 0$, so (1) reduces to

$$L_1 = \prod_x \prod_y [\text{pr}(R = 0 | Z = 0, x; \zeta) \text{pr}(y | Z = 0, x; \pi)]^{n_{0xy}} \prod_x [\sum_y \text{pr}(R = 1 | Z = 1, x; \zeta) \text{pr}(y | Z = 1, x; \pi)]^{w_{1x}}. \quad (2)$$

From (2), the part of the likelihood involving π is $L_1^* = \Pi_x \Pi_y [\text{pr}(y | Z = 0, x; \pi)]^{n_{0xy}}$, which only corresponds to subjects in Phase 2. Because $Z = 0$ in L_1^* not all parameters can be estimated. For purposes of illustration, it is helpful to consider a particular model, $\text{logit}(\text{pr}(y | z, x; \pi)) = \pi_0 + \pi_X x + \pi_Z z +$

$\pi_{XZ}xZ$, where $\pi = (\pi_0, \pi_X, \pi_Z, \pi_{XZ})$. In L_1^* the model is $\text{logit}(\text{pr}(y | Z = 1, x; \pi)) = \pi_0 + \pi_X x$. Thus it is not possible to estimate π_Z or π_{XZ} . In other words, using the formulation in (2), we can only make inference about the effect of balance on depression for subjects in Phase 2. The goal is make inference for all subjects. To circumvent this difficulty we formulated a new likelihood kernel with an ignorable missing-data mechanism,

$$L_2 = \prod_x \prod_y [\text{pr}(R = 0 | Z = 0, x; \tau) \text{pr}(Z = 0 | y, x; \eta) \text{pr}(y | x; \beta)]^{n_{1xy}} \prod_x [\sum_y \text{pr}(R = 1 | Z = 1, x; \tau) \text{pr}(Z = 1 | y, x; \eta) \text{pr}(y | x; \beta)]^{w_{0x}}, \quad (3)$$

where β is the parameter of interest that applies to all subjects. We cannot reduce (3) to $\prod_x \prod_y [\text{pr}(y | x; \beta)]^{n_{1xy}}$ because of the dependence of Z on y . Because $\text{pr}(Z = 0 | y, x; \eta) = \text{pr}(R = 0 | y, x; \eta)$ and $\text{pr}(Z = 1 | y, x; \eta) = \text{pr}(R = 1 | y, x; \eta)$ (3) reduces to the likelihood kernel

$$L_3 = \prod_x \prod_y [\text{pr}(R = 0 | y, x; \eta) \text{pr}(y | x; \beta)]^{n_{xy}} \prod_x [\sum_y \text{pr}(R = 1 | y, x; \eta) \text{pr}(y | x; \beta)]^{w_x}, \quad (4)$$

where $n_{xy} = n_{1xy}$ and $w_x = w_{0x}$. If $\text{pr}(R = r | y, x; \eta) = \text{pr}(R = r | x; \eta)$ and η is distinct from β , the missing-data mechanism in (4) is ignorable; otherwise it is nonignorable.

Elashoff and Elashoff (1974); Fay (1986); Baker and Laird (1988) independently developed the first models to adjust for nonignorable missing-data mechanisms with categorical data. These nonignorable missing-data models are identifiable when some higher-order interactions are set to zero. For the simple case of a binary covariate and a binary outcome, as in Table 1, Baker and Laird (1988) derived closed-form maximum likelihood estimates when missingness depends on the outcome but not on the covariate. Maximum likelihood estimates are obtained by solving two equations in two parameters, unless the solution is on the boundary of the parameter space. Using the simple algorithm in Baker and Laird (1988) we determined that the solution for the data in Table 1 was on the boundary of the parameter space. Using the closed-form boundary estimates in Baker and Laird (1988) with variance computation via the MP-transformation (Baker, 1994c), we estimated the logarithm of the odds ratio as 2.8 with an asymptotic standard error of 0.04. However, the large deviance of 101 on zero degrees of freedom indicated a poor fit and suggested that additional covariates were needed.

With nonignorable missing-data mechanisms, results from a simple model with one covariate can change considerably with the addition of more covariates. For a single covariate with an interior solution, the odds ratio for the effect of covariate on outcome and its standard errors are identical for ignorable and nonignorable models (Elashoff and Elashoff, 1974; Baker and Laird, 1988; Baker *et al.*, 1992). With even one additional covariate, the odds ratios under ignorable and nonignorable missing-data models can differ even if the probability of missing in the latter model depends on outcome and not also the interaction between outcome and one of the covariates.

Missing in both depression and balance

Extending (4), we also formulated a likelihood to account for the 2% of subjects missing in balance. Let $S = 1$ if a subject was missing in balance and 0 otherwise. Because missingness in depression occurs after missingness in balance, we allow the probability of missing in depression to depend on S . Let u_y denote the number of subjects with depression outcome y who are missing balance. Let v denote the number of

subjects missing both balance and depression. The likelihood kernel is

$$\begin{aligned}
L_4 = & \prod_x \prod_y [\text{pr}(R = 0 \mid S = 0, y, x; \psi) \text{pr}(S = 0 \mid y, x; \theta) \text{pr}(y \mid x; \beta) \text{pr}(x \mid \alpha)]^{n_{xy}} \\
& \prod_x [\sum_y \text{pr}(R = 1 \mid S = 0, y, x; \psi) \text{pr}(S = 0 \mid y, x; \theta) \text{pr}(y \mid x; \beta) \text{pr}(x \mid \alpha)]^{w_x} \\
& \prod_y [\sum_x \text{pr}(R = 0 \mid S = 1, y, x; \psi) \text{pr}(S = 1 \mid y, x; \theta) \text{pr}(y \mid x; \beta) \text{pr}(x \mid \alpha)]^{u_y} \\
& [\sum_x \sum_y \text{pr}(R = 0 \mid S = 1, y, x; \psi) \text{pr}(S = 1 \mid y, x; \theta) \text{pr}(y \mid x; \beta) \text{pr}(x \mid \alpha)]^v. \quad (5)
\end{aligned}$$

Unlike the case with missing in only one variable, if $\text{pr}(R = r \mid s, y, x; \psi) = \text{pr}(R = r \mid x; \psi)$ or $\text{pr}(S = s \mid x; \theta) = \text{pr}(S = s \mid y, x; \theta)$ the missing data mechanism is not ignorable. This is an example of a type II nonignorable missing-data mechanism, where missingness in a variable does not depend on that variable but depends on one or more other variables that are partially missing (Baker, 2000). In contrast, a type I nonignorable missing-data mechanism means that missingness in a variable depends on the partially observed value of the variable and may or may not depend on other partially observed variables. A type I nonignorable missing-data mechanism represents a conceptually different type of missing-data mechanism and is the primary focus of our missing-data adjustment.

Missing in depression with an auxiliary variable

Extending (4), we also formulated a likelihood that incorporates an auxiliary variable, which we define as a completely observed variable that occurs after baseline and is strongly related to the partially observed outcome variable. The auxiliary variable provides information on the outcome variable when the outcome variable is not observed. Here the auxiliary variable is a response to a Phase 1 question on frequent depression or anxiety, which likely has a strong association with the outcome variable: medically treated depression. In fact, all subjects with medically treated depression in Phase 2 also answered yes to depression or anxiety in Phase 1. To avoid added computational difficulties, particularly with the sparse data, we discarded data from the 2% of subjects missing in balance. Let A denote the binary auxiliary variable. The likelihood kernel is

$$\begin{aligned}
L_5 = & \prod_a \prod_x \prod_y [\text{pr}(R = 0 \mid a, y, x; \delta) \text{pr}(a \mid y, x; \lambda) \text{pr}(y \mid x; \beta)]^{n_{axy}} \\
& \prod_a \prod_x [\sum_y \text{pr}(R = 1 \mid a, y, x; \delta) \text{pr}(a \mid y, x; \lambda) \text{pr}(y \mid x; \beta)]^{w_{ax}}, \quad (6)
\end{aligned}$$

where n_{axy} is the number of subjects with auxiliary variable at level a , balance at level x and observed depression outcome y ; and w_{ax} is the number of subjects missing the depression outcome who have auxiliary variable at level a and balance at level x . If $\text{pr}(R = r \mid a, y, x; \delta) = \text{pr}(R = r \mid a, x; \delta)$ and δ is distinct from λ and β ; the missing-data mechanism is ignorable, otherwise it is nonignorable. In analysing data for a missing outcome in a randomized trial, Baker (2000) formulated (6) for an ignorable missing-data mechanism with binary covariate, auxiliary variable and outcome. The closed-form solution in Baker (2000) is a simple function of the observed counts and the imputed counts, where the imputation is based on the covariate and the auxiliary variable. In analysing observational data with a missing covariate, Horton and Laird (2001) and Ibrahim *et al.*, (2001) also formulated a related likelihood under an ignorable missing-data mechanism.

The likelihood kernel with an auxiliary variable in (6) can be viewed as a generalization of the likelihood kernel without an auxiliary variable in (4). We investigated (6) and (4) separately because

they involved very different missing-data models and, as part of our sensitivity analysis, we wanted to investigate how the results varied. In theory the likelihood kernel for missing in two variables in (5) could also be generalized to include an auxiliary variable, but the computations are difficult.

3. MODELS

We used the following general approach to model selection. First we specified separate ignorable missing-data mechanisms with main effects, two-way interactions and three-way interactions. Second, for each of these ignorable missing-data mechanisms, we specified a nonignorable missing-data mechanism by adding an interaction between missingness in outcome and outcome. For further investigation, we subsequently added an interaction between missingness in outcome, outcome, and balance. The reason for starting with ignorable missing-data mechanisms in this likelihood-based approach was robustness. All ignorable missing-data mechanisms give rise to the same estimates and standard errors for parameters in the model of interest. The rationale for perturbing multiple ignorable missing-data mechanisms was to investigate the sensitivity of the perturbation to the initial model from the interaction of missingness in outcome with outcome. The investigation of multiple missing-data mechanisms is a key feature of the sensitivity analysis.

Missing only in depression

We initially discarded data from the 2% of subjects missing balance and considered the computationally simpler case of missingness only in depression. We let X_{balance} equal 1 if there was chronic imbalance/dizziness and 0, otherwise; X_{age} indicates age categories 18 to 44, 45 to 64, and 65 or older; X_{race} indicates white, black, or other; X_{gender} indicates men or women, X_{health} indicates self-described health status, either poor to fair or good to excellent; X_{work} indicates working or not working; and Y equals 1 if frequent medically treated depression and 0 otherwise. Also we let R equal 1 if missing depression and 0 otherwise. We specified the model of interest as

$$\begin{aligned} \text{logit}(\text{pr}(Y = 1)) = & \beta_0 + \beta_B X_{\text{balance}} + \beta_A X_{\text{age}} + \beta_G X_{\text{gender}} \\ & \beta_R X_{\text{race}} + \beta_H X_{\text{health}} + \beta_W X_{\text{work}}, \end{aligned} \quad (7)$$

where the parameter of interest is β_B , the regression coefficient for the effect of balance problems on frequent depression. For our sensitivity analysis, we started with three ignorable missing-data models: a main effects model,

$$\begin{aligned} \text{logit}(\text{pr}(R = 0)) = & \eta_0 + \eta_B X_{\text{balance}} + \eta_A X_{\text{age}} + \eta_G X_{\text{gender}} \\ & + \eta_R X_{\text{race}} + \eta_H X_{\text{health}} + \eta_W X_{\text{work}}, \end{aligned} \quad (8)$$

and hierarchical models with two- and three-way interactions. To construct nonignorable missing-data models, we added $\eta_D Y$, and subsequently $\eta_{B^*D} X_{\text{balance}} Y$ to (8) and its extensions.

Missing in balance and depression

Because of computational limitations and sparse data, we needed to drop one covariate. We dropped X_{work} because, of all the covariates, it had the weakest association with participation in Phase 2. We specified the same model of interest as in (7) but without X_{work} . With standard hierarchical parametrizations for the missing-data mechanisms, the only ignorable missing-data mechanism is missing completely at random (Robins and Gill, 1997), which is too limited for our purposes. To create a rich class of ignorable

missing-data models, we used a parametrization similar to Baker (1996), which is a type of randomized monotone missingness process (Robins and Gill, 1997). For the probability that balance was not missing, we specified the following ignorable missing-data model:

$$\text{logit}(\text{pr}(S = 0)) = \theta_0 + \theta_G X_{\text{gender}} + \theta_A X_{\text{age}} + \theta_R X_{\text{race}} + \theta_H X_{\text{health}}, \quad (9)$$

and hierarchical models with two-way and three-way interactions. To construct nonignorable missing-data models for missing in balance, we added $\theta_B X_{\text{balance}}$ to (9) and its extensions. For the probability that depression was not missing, we specified the following ignorable missing-data model:

$$\begin{aligned} \text{logit}(\text{pr}(R = 0)) = & (\psi'_0 + \psi'_G X_{\text{gender}} + \psi'_A X_{\text{age}} + \psi'_R X_{\text{race}} + \psi'_H X_{\text{health}})S \\ & + (\psi_0 + \psi_B X_{\text{balance}} + \psi_G X_{\text{gender}} + \psi_A X_{\text{age}} + \psi_R X_{\text{race}} + \psi_H X_{\text{health}})(1 - S), \end{aligned} \quad (10)$$

and similar hierarchical models with two-way and three-way interactions, the latter restricted to interactions with balance to minimize problems with sparse data. To construct nonignorable missing-data models, we added $\psi_D Y$, and subsequently $\psi_{BD} X_{\text{balance}} Y$ to (10) and its extensions.

Missing in depression with auxiliary variable

Because of computational limitations and sparse data, we needed to drop one covariate, which we arbitrarily selected as X_{work} . We specified the same model as in (7) but without X_{work} . For simplicity we also specified the following model relating the auxiliary variable to the other covariates:

$$\text{logit}(\text{pr}(A = 1)) = \lambda_0 + \lambda_G X_{\text{gender}} + \lambda_A X_{\text{age}} + \lambda_R X_{\text{race}} + \lambda_H X_{\text{health}}. \quad (11)$$

For the probability that depression was not missing, we specified the following ignorable missing-data model:

$$\text{logit}(\text{pr}(R = 0)) = \delta_0 + \delta_G X_{\text{gender}} + \delta_A X_{\text{age}} + \delta_R X_{\text{race}} + \delta_H X_{\text{health}} + \delta_{\text{aux}} A, \quad (12)$$

and hierarchical models with two- and three-way interactions, the latter restricted to interactions with balance to minimize problems with sparse data. To construct nonignorable missing-data models, we added $\delta_D Y$ and subsequently $\delta_{BD} X_{\text{balance}} Y$ to (12) and its extensions.

4. ESTIMATION

In analysing these data a major consideration was the complex sample survey design. The 1994 and 1995 NHISs are cross-sectional household surveys with multistage stratified cluster area probability samples. The first stage of sampling involved the selection of primary sampling units (PSUs), consisting of counties or metropolitan areas. For purposes of variance estimation, the 1994 NHIS sample design was approximated by the sampling of four pseudo-PSUs from 62 pseudo-strata. Pseudo-strata and pseudo-PSUs are modifications of the original sampling strata and PSUs that preserve confidentiality, facilitate variance computation, and can be treated as usual strata and PSUs for variance estimation (Korn and Graubard, 1999). The 1995 NHIS sample design was approximated by the sampling of two pseudo-PSUs from 187 pseudo-strata. Because NHIS was redesigned in 1995 with a new set of strata and sample of PSUs, the 1994 and 1995 NHISs were treated as independent samples for estimating variances in our analyses, which combined the two samples of data. The sample design for the combination of the two surveys was approximated by 249(= 62 + 187) pseudo-strata with four or two pseudo-PSUs depending

upon which survey year the strata came from. The sample weights, as assigned to each sampled individual on the public use files, were used to estimate weighted cell counts in the analyses.

Estimation is based on a weighted likelihood in which observations were weighted according to the sample weights. For a general discussion of weighted likelihoods see Skinner *et al.* (1989). We maximized the weighted likelihood involving missing categorical data using the composite linear model approach (Baker, 1994a) which specifies a matrix EM algorithm and then switches to a matrix Newton–Raphson algorithm. See <http://dcp.nci.nih.gov/bb>. To avoid numerical problems, we added 0.05 to cells with zero counts. Starting with the EM algorithm was important for obtaining good starting values for the Newton–Raphson algorithm. It is not surprising that Bonetti *et al.* (1999) reported problems with only using a Newton–Raphson algorithm to fit type I nonignorable missing-data models.

Although there have been various papers on fitting nonignorable missing-data models to survey data (Stasny, 1987, 1988, 1990; Conaway, 1992, 1993; Chambers and Welsh, 1993; Forster and Smith, 1998; Heitjan and Landis, 1994), they did not fully account for the complex survey design. In contrast, to account for the stratification and cluster sampling, we computed variances using a first-order Taylor series linearization (e.g. Korn and Graubard, 1999), which is a delta method that gives rise to a sandwich estimator (see the Appendix). To facilitate computation we use matrices in the composite linear framework. However, instead of a single vector of counts over all cross-classifications of variables, the input is a vector of counts for each of the 622 (= $62 \times 4 + 187 \times 2$) PSUs. As a check of the linearization variance formula, for some of the models we also computed standard errors using the jackknife method for sample surveys (e.g. Korn and Graubard, 1999). The agreement was excellent. The advantage of the linearization variance over the jackknife approach is that the computations were substantially faster.

5. RESULTS AND DISCUSSION

For a sensitivity analysis with missing categorical data, three common strategies are (i) fit all possible saturated nonignorable missing-data models (e.g. Baker *et al.* (1992); Robins (1997), for type II nonignorable missing-data mechanisms, Molenberghs *et al.* (2001)), (ii) plot the estimated parameter of interest as a function of a key missing-data parameter (e.g. Vach and Blettner (1995)) and (iii) fit various ignorable missing-data models, add one or two parameters to make the missing-data model nonignorable, and compare goodness of fit and estimates (e.g. Baker and Laird 1988; Baker, 1994a,b, 1995a,b, 1996; Fitzmaurice *et al.*, 1996).

To our knowledge, strategy (i) has only been applied in problems with few covariates; generalization to many covariates is a topic for future research. Strategy (ii) is most useful when there is prior knowledge of likely values for the effect of outcome on the probability of missing. Strategy (iii) has the important advantage of using information on goodness of fit to sharpen inference. However, strategy (iii) is generally only useful with large sample sizes and many covariates. As illustrated in Table 2, when strategy (iii) is applied to small data sets with few covariates, the typical result (with Baker (1995b) a notable exception) is either (a) little change in deviance between ignorable and nonignorable missing-data models, (b) a high degree of overlap in confidence intervals for estimates of interest under ignorable and nonignorable missing-data mechanisms, or (c) boundary solutions, that likely indicate a misspecified model. Because we had very large sample sizes and many covariates, strategy (iii) was especially helpful for our analysis.

Using strategy (iii) we found that adding one or two parameters to make the missing-data mechanism for depression nonignorable dramatically decreased the deviance in a variety of models (Tables 3–5). For example, in Table 3 with two-way interactions in missingness in depression, the change of deviance was 40 on one degree of freedom. Although the deviance only approximately follows a chi-square distribution

Table 2. Type I nonignorable missing-data models for categorical data

Study	Size	Type of study	Number of variables	Missing in	Type [†]	Fraction missing at least one variable(%)		ML estimate (s.e.)		Change in deviance (d.f.)
						ignorable	nonignorable	ignorable	nonignorable	
Fay (1986)	164	survey	3	outcomes	S	38	0.890 (0.296)	0.724 (0.318)	4 (3)	
Stasny (1987)	24 012	survey/flows	2	outcomes	S	24	0.9435	0.9436		
Stasny (1988)	138 697 ^o	survey/flows	2	outcomes	S	12	3599 (80)	3621 (81)	230 (1)	
Baker and Laird (1988)	13 491	survey	3	outcome	S	17	0.413 (0.009)*	0.517 (0.008)*	17 (1)	
Stasny (1990)	12 432	survey/flows	2	outcomes	S	25	0.013 (0.026)	0.015 (0.008)	32 (2)	
Conaway (1992)	133 629	survey/flows	2	outcomes	S	12	0.24 1.75	410 (2)		
Conaway <i>et al.</i> (1992)	2 082	survey	3	cov, outcome	L	59	0.164 (0.066)*	0.295 (.099)*	34 (1)	
Conaway (1993)	4 723	survey/flows	2	outcomes	S	47	0.804 0.760	95 (6)		
Baker <i>et al.</i> (1992)	57 061	cohort	2	outcomes	L	7	0.352 (0.032)*	0.332 (.058)*	< 1 (1)	
Baker (1994a)	90	experiment	2	margins	S	12	0.201 (0.031)	0.252 (.031)	< 1 (0)	
Baker (1994b)	79	survival	2	covariates	S	52	1.40 (0.51)	1.48 (.54)*	< 1 (1)	
Baker (1995a)	1 788	diagnostic	4	test result	S	59	0.208 (0.101)*	0.578 (.055)	3 (1)	
Baker (1995b)	4 856	longitudinal	5	outcomes	S	69	0.069 (0.007)*	0.015 (.005)	106 (3)	
Baker (1996)	814	case-control	3	exposures	S	20	-0.51 (0.15)*	-0.53 (.15)	4 (1)	
Ibrahim and Lipsitz (1996)	3 369	cohort	3	outcome	S	38	0.279 (0.116)	0.098 (.136)		
Fitzmaurice <i>et al.</i> (1996)	2 501	survey	4	outcomes	S	43	0.648 (0.185)	0.643 (.363)	< 1 (1)	
Molenberghs <i>et al.</i> (1997)	299	cohort	5	outcome	S	18	-0.19 (0.07)	-0.23 (.06)	4 (1)	
Forster and Smith (1998)	1 242	survey	3	outcome	S	30	0.35 (0.02)**	0.32 (.06)**		
Park (1998)	109	cohort	3	outcome	S	43	0.330 (0.072)*	0.605 (.056)*	3 (1)	
Verbeke <i>et al.</i> (2001)	286	clinical trial	7	covariates	S	29	0.721 (0.260)	0.717 (.259)		
Molenberghs <i>et al.</i> (1999)	315	cohort	2	outcomes	L	29	1.96 (0.36)	1.94 (.37)	< 1 (0)	
Raab and Donnelly (1999)	2 308	survey	3	outcomes	S	38	-0.71 (0.18)**	-0.95 (.36)**		
Albert (2000)	108	longitudinal	5	outcomes	S	80	2.23	2.31 (.017)		
Paik <i>et al.</i> (2000)	294	cohort	7	outcomes	S	13	-0.62 (0.30)	-0.71 (.29)		
Verbeke <i>et al.</i> (2001)	50	trial	2	outcome	S	56	7.31 (0.28)	7.35 (.30)	< 1 (0)	
Molenberghs <i>et al.</i> (2001)	2 074	survey	2	outcomes	L	25	0.891 (0.007)**	0.779 (.038)**	27 (1)	
current study	145 007	survey	7	cov, outcome	S	82	0.68 (0.06)	1.03 (.05)	33 (1)	

[†] model type is S for selection model and type L for loglinear model.

^{††} wherever possible the estimate corresponds to primary parameter; otherwise a typical estimate is reported.
^o excluding rotation

* standard errors and/or estimates not reported in paper but computed with CLM software.

** standard errors approximated from confidence interval

Table 3. *Analysis of health survey with missing only in depression*

Missing-data mechanism**		d.f.	Deviance	Effect of balance on depression*	Effect of depression on the probability of missing depression*
main effects	ignorable	271	2568	0.68 (0.04)	
	nonignorable D	270	2201	1.23 (0.05)	2.98 (0.27)
	nonignorable D+BD	269	2164	1.04 (0.06)	
two-way	ignorable	245	517	0.68 (0.04)	
	nonignorable D	244	477	1.01 (0.07)	1.33 (0.22)
	nonignorable D+BD	243	477	1.04 (0.17)	
three-way	ignorable	201	367	0.68 (0.04)	
	nonignorable D	200	333	1.00 (0.07)	1.28 (0.24)
	nonignorable D+BD	199	328	1.41 (0.21)	

*estimated coefficient in logistic regression with standard errors in parentheses.

**The model nonignorable D adds one parameter to the ignorable missing-data model to allow missing in depression to depend on depression. The model nonignorable D+DB also adds a parameter to allow missing in depression to depend on the interaction between balance and depression.

due to the complex survey design, this large change in deviance indicates a substantial improvement in fit. Importantly, a large sample size does not guarantee a large change in deviance. When analysing the same data but with experiencing falls as the outcome (not shown), we found a difference in deviance of less than 1 between an ignorable and nonignorable model with two-way interactions. Hence, we believe that the nonignorable missing-data models are very informative. Therefore, for those models, we report a range of odds ratios for balance and depression. Under 29 of the 30 selected nonignorable missing-data models, the odds ratios ranged from 2.7 with 95% CI of (2.3, 3.1) to 4.2 with 95% CI of (3.9, 4.6). Under one nonignorable missing-data model, the odds ratio was 7.4 with 95% CI of (6.3, 8.6). Because a saturated ignorable missing-data model will fit as well or better than a nonignorable missing-data mechanism, it is also important to report the estimates under the ignorable missing-data mechanism, namely an odds ratio of 2.0 with 95% CI of (1.8, 2.2). Despite the fact that the missing-data mechanisms varied substantially among the models, the results from all the models indicated a strong association between balance difficulties and frequent medically treated depression with lower bounds on the 95% confidence intervals of at least 1.8.

In all the models for the nonignorable missing-data mechanism (except with the auxiliary variable which complicates interpretation), the probability of being observed in depression increased as a function of depression (e.g. a regression coefficient of 2.98 with standard error of only 0.27), as expected. This agrees with our view that depressed subjects were most likely included in Phase 2.

Applying strategy (ii) we plotted the odds ratio for balance and depression (with 95% confidence interval) as function of the regression coefficient for the effect of depression on the probability of missing depression (Figures 1 and 2), which corresponds to parameter η_D , ψ_D , or δ_D , associated with (8), (10) and (12) respectively. This is the type of plot proposed by Vach and Blettner (1995). Because goodness of fit is a consideration, we also plotted the profile deviances as a function of the regression coefficient for the effect of depression on the probability of missing depression. In the model involving missing in both balance and depression with main effects (Figure, 2 bottom), there were two local minima in the range of plausible values for ψ_D . In all other models, we found only one local minimum in the range of plausible values. The plots for the auxiliary variable model differed considerably from those of the other

Table 4. Analysis of health survey with missingness in balance and depression

Missing-data mechanism**		d.f.	Deviance	Effect of balance on depression*	Effect of variable on missingness in variable
main	ignorable:	222	1381	0.68 (0.04)	
	nonignorable in balance	221	1381	0.68 (0.04)	0.76 (0.85)
	nonignorable in depression D	221	1321	1.08 (0.05)	1.51 (0.17)
	nonignorable in depression D+DB	220	1053	2.00 (0.08)	
two-way	ignorable	177	330	0.68 (0.04)	
	nonignorable in balance	176	330	0.68 (0.04)	0.60 (1.63)
	nonignorable in depression D	176	297	1.03 (0.06)	1.24 (0.19)
	nonignorable in depression D+DB	175	295	1.36 (0.29)	
three-way†	ignorable	140	278	0.68 (0.04)	
	nonignorable in balance	139	278	0.68 (0.04)	0.76 (0.85)
	nonignorable in depression D	139	245	1.03 (0.06)	1.26 (0.19)
	nonignorable in depression D+DB	138	241	1.43 (0.23)	

*estimated coefficient in logistic regression with standard errors in parentheses.

** the model nonignorable in balance adds one parameter to the ignorable model to allow missing in balance depend on balance. The model nonignorable in depression D adds one parameter to the ignorable missing-data model to allow missing in depression to depend on depression. The model nonignorable in depression D+DB also adds a parameter to allow missing in depression depend on the interaction between balance and depression.

†to avoid numerical problems with sparse data, the three-way interactions were included only when balance was observed.

Table 5. Analysis of health survey with missing only in depression and auxiliary variable

Missing-data mechanism**		d.f.	Deviance	Effect of balance on depression*	Effect of depression on the probability of missing depression*
main effects	ignorable	334	3674	1.44 (0.04)	
	nonignorable D	333	3094	1.24 (0.05)	-5.8 (0.09)
	nonignorable D+BD	332	2373	1.09 (0.05)	
two-way	ignorable	315	2835	1.44 (0.04)	
	nonignorable D	314	2194	1.26 (0.05)	-5.8 (0.09)
	nonignorable D+BD	313	1697	1.23 (0.04)	
three-way†	ignorable	302	2803	1.44 (0.04)	
	nonignorable D	301	2166	1.26 (0.05)	-5.8 (0.09)
	nonignorable D+BD	300	1671	1.23 (0.04)	

*estimated coefficient in logistic regression with standard errors in parentheses.

** the model nonignorable D adds one parameter to the ignorable missing-data model to allow missing in depression to depend on depression. The model nonignorable D+BD also adds a parameter to allow missing in depression depend on the interaction between balance and depression.

†to avoid numerical problems with sparse data, the three-way interactions were included only when balance was observed

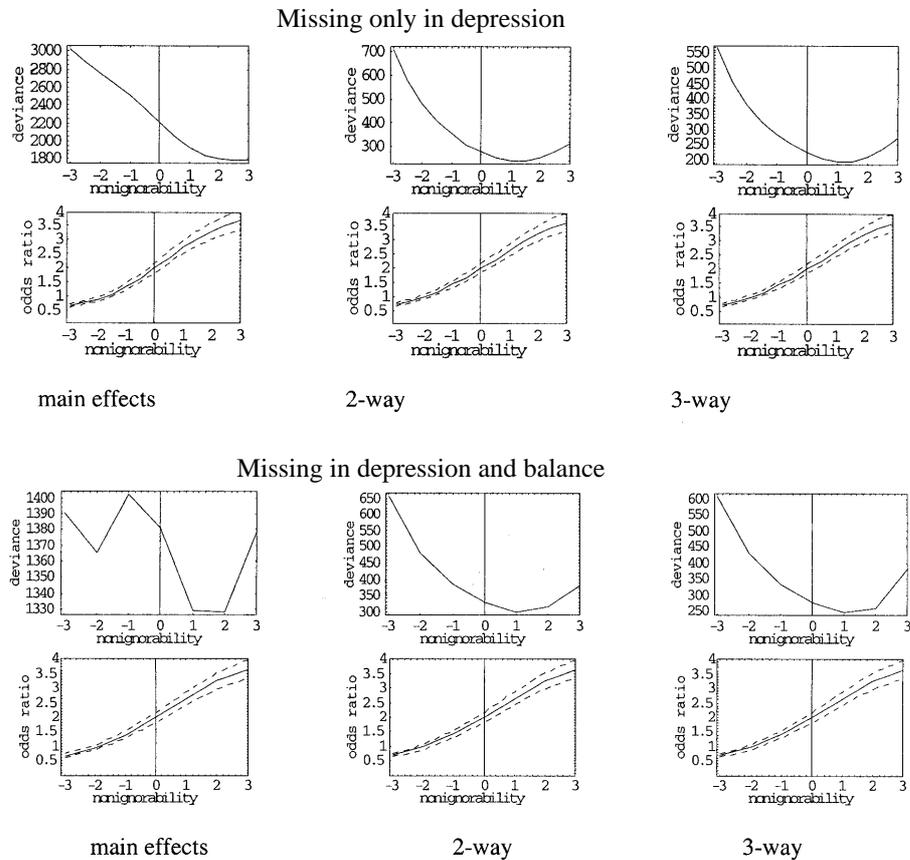


Fig. 1. Effect of Missing in Depression on Deviance and Odds Ratio. The odds ratio in the lower plot refers to balance and medically treated depression, with the dashed lines indicating 95% confidence intervals. Nonignorability refers to the logarithm of the odds ratio for depression and missingness in depression. A value of 0 indicates an ignorable missing-data model. A value of 3 corresponds to an odds ratio of 20 and a value of -3 corresponds to an odds ratio of $1/20$. A value between 1 and 3 corresponds to the minimum deviance; values farther away give rise to extremely large changes in deviances.

models because missingness also depended on the auxiliary variable. For the auxiliary variable models, we experienced computational problems for λ_D less than the value at the minimum deviance. This was likely due to the very sparse data and the fact the λ_D at the minimum deviance was so small. The large odds ratios for acceptable values of the deviances confirm the strong association between balance and depression over all models and reasonable ranges of η_D , ψ_D , and λ_D .

Our sensitivity analysis has important consequences for public health. This analysis was the first investigation of the relationship between psychological outcome and chronic balance or dizziness problems in the overall adult population of the United States. Previous studies of balance and depression in more specialized populations found either no association or a weak association. In contrast, for all models that we investigated we found a very large association between balance and medically treated depression.

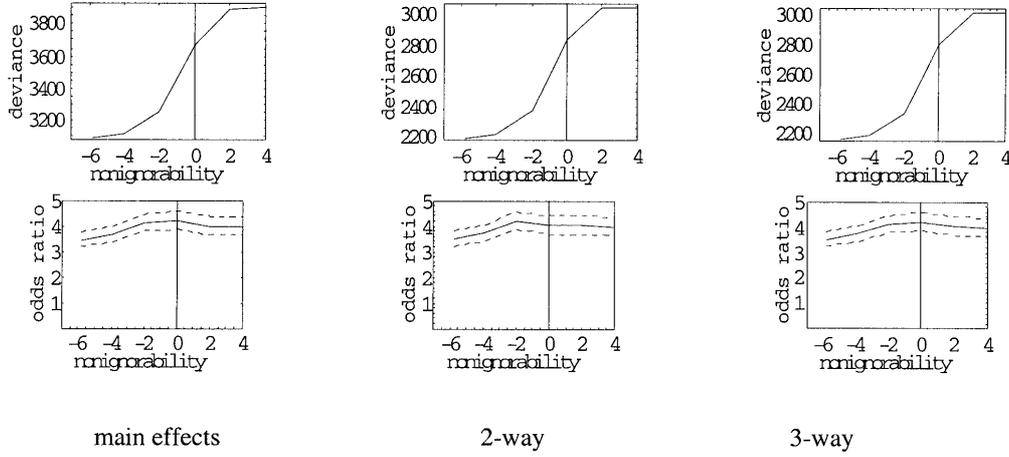


Fig. 2. Effect Missing in Depression on Deviance and Odds Ratio With Auxiliary Variable. The odds ratio in the lower plot refers to balance and medically treated depression, with the dashed lines indicating 95% confidence intervals. Nonignorability refers to the logarithm of the odds ratio depression and missingness in depression within a model in which the missingness in depression also depends on the auxiliary variable. A value of 0 indicates an ignorable missing-data model. A value near -6 corresponds to the minimum deviance; calculations with smaller values exhibited computational problems.

APPENDIX: LINEARIZATION VARIANCE

Let s index stratum and $c = 1, \dots, n_s$ index PSU within stratum s . Also let j index a cross-classification of all the variables and y_{scj} denote the sum of weights over all individuals in stratum s , PSU c , and cross-classification j . Let $Y_{sc} = \{y_{sc1}, \dots, y_{scj}\}$, $DIF_{sc} = Y_{sc} - \sum_c Y_{sc}/n_s$ and $Y = \sum_s \sum_c Y_{sc}$. Following the notation in Baker (1994a), let $C = \{c_{ij}\}$ be the matrix mapping the complete to incomplete data. Given the design matrices and the functional forms, the software automatically computes $U =$ a vector of expected counts for the incomplete data Y , $U^* = \{u_i^*\} =$ vector of expected counts for the complete data, $S = \{s_{ja}\} =$ vector used in computing the score statistic, and ObsInf , the observed information matrix. The linearization variance is $\text{ObsInf}^{-1} \text{Core} \text{ObsInf}^{-1}$, where

$$\text{Core} = \sum_s \sum_{c=1}^{n_s} \frac{n_s}{n_{s-1}} K DIF_{sc} DIF'_{sc} K',$$

and $K = \text{diag}(1/U)C \text{diag}(U^*)S$. The derivation of Core follows that in Baker (1994a). Element a of the score vector is $\text{Score}_{sca} = \sum_j y_{scj} k_{ja}$ where $k_{ja} = (\sum_i c_{ji} u_i^* s_{ia})/u_j$ as in Baker (1994a). The mean score for element a is $\text{MeanScore}_{sa} = \sum_c \text{Score}_{sca}/n_c$. The (a, b) element of Core is thus

$$\begin{aligned} \text{Core}_{ab} &= \sum_s \frac{n_s}{n_{s-1}} \sum_c (\text{Score}_{sca} - \text{MeanScore}_{sa})(\text{Score}_{scb} - \text{MeanScore}_{sb}) \\ &= \sum_s \sum_{c=1}^{n_s} \frac{n_s}{n_{s-1}} \left[\sum_{j=1}^{n_{sc}} (y_{scj} - \sum_{c=1}^{m_c} y_{scj}/n_{sc}) k_{ja} \sum_{j=1}^{n_{sc}} (y_{scj} - \sum_{c=1}^{m_c} y_{scj}/n_{sc}) k_{jb} \right] \\ &= \sum_s \sum_{c=1}^{n_s} \frac{n_s}{n_{s-1}} \left[\sum_{j=1}^{n_{sc}} DIF_{scj} k_{ja} \right] \left[\sum_{j=1}^{n_{sc}} DIF_{scj} k_{jb} \right] \end{aligned}$$

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